ground water

Stream Depletion in Alluvial Valleys Using the SDF Semianalytical Model

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Abstract

A semianalytical method commonly used for quantifying stream depletion caused by ground water pumping was reviewed for applicability in narrow alluvial aquifers. This stream depletion factor (SDF) method is based on the analytic Glover model, but uses a numerical model–derived input parameter, called the SDF, to partly account for mathematically nonideal conditions such as variable transmissivity and nearby aquifer boundaries. Using the SDF can improve and simplify depletion estimates. However, the method's approximations introduce error that increases with proximity to the impermeable aquifer boundary. This article reviews the history of the method and its assumptions. New stream depletion response curves are presented as functions of well position within bounded aquifers. A simple modification to modeled SDF values is proposed that allows the impermeable boundary to be accounted for with image wells, but without overaccounting for boundary effects that are already reflected in modeled SDFs. It is shown that SDFs for locations closer to the river than to the aquifer boundary do not reflect impermeable-boundary effects, and thus need no modification, and boundary effects in the other portion of the aquifer follow a predictable removable pattern. This method is verified by comparing response curves using modified SDFs with response curves from an extensively calibrated numerical model of a managed ground water recharge site. The modification improves SDF-based stream depletion estimates in bounded aquifers while still benefiting from the additional information contained in SDF maps and retaining their value as standardized references for water rights administration.

Introduction and History

Conjunctive regulation of ground water and surface water has long been practiced in the semiarid western United States. All 13 western states that use the "prior appropriation doctrine" to govern water rights have "a legal foundation for integrated management of hydrologically connected surface water and groundwater" (Grant 1987), though in many western states the regulatory programs are still developing or evolving. Even eastern states have begun to assess the impact of ground water pumping on streams (e.g., Mueller and Male 1993; Zarriello and Reis 2000). Regulatory agencies frequently use or specify the "Glover method" (Glover and Balmer 1954), the "stream depletion factor (SDF) method" (Jenkins 1968a, 1970), or an "equivalent" method for quantifying such impacts. These models compute transient stream depletion induced by ground water pumping from hydrologically connected aquifers. They are also used for assessing stream accretions from managed ground water recharge. The discussions and results herein apply to either case.

The SDF is an alterative input to the equation of Glover and Balmer (1954) and was defined by Jenkins (1970) as the time when stream depletion is equal to 28% of the volume pumped for a given location. The SDF definition is frequently misunderstood and will be discussed further in a following section. The U.S. Geological Survey (USGS) used modeling to map SDF values for

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Received May 2006, accepted January 2007.

doi: 10.1111/j.1745-6584.2007.00311.x

the alluvial aquifers along the South Platte and Arkansas rivers in Colorado (e.g., Hurr and Schneider 1972a, 1972b). An example map is shown in Figure 1.

In 1974, a water court in Colorado approved "Amended Rules and Regulations for the South Platte," which state that stream depletions caused by a well should be calculated using the "Glover formula or by other accepted engineering formulae appropriately modified to reflect the pertinent physical conditions" (Water Division No. 1 1974). MacDonnell (1988) notes that the ground water augmentation plan developed by a local reservoir and irrigation company in 1985 used the SDF to analyze depletions, accretions, and the "net stream effect" and that "the water court essentially adopted the analytical approach." Statutes revised in 2003 for certain water supply plans required use of "the United States Geological Survey stream depletion factor method for all areas covered by such factors" (Colorado Revised Statutes §37-92-308 3(c)(II) 2005). For much of the last 30 years, the predominant method used to compute ground water pumping and recharge effects on the South Platte River was the SDF method using USGS SDF maps (Warner et al. 1994; Cuthbertson 2005).

Additional SDF modeling has been performed in the Platte and Kansas river basins in Wyoming, Nebraska, and Kansas as part of a Missouri River basin hydrology study (Missouri Basin States Association 1982). These maps plus other SDF modeling efforts have been used in new regulatory programs in Nebraska (COHYST Technical Committee 2004; Nebraska Dept. of Natural Resources 2005) and have been partly reissued in electronic geographical information system (GIS) format (cohyst.dnr.ne.gov). Detailed GIS SDF maps for the South Platte River in Colorado are available through Colorado State University (www.ids.colostate.edu). Regulatory agencies in several other states also cite Jenkins (1968a) as guidance for stream depletion assessments.

Despite many recent advancements in analytical solutions for stream depletion analysis and today's numerical modeling capacity, it is likely that the simpler and established Glover and SDF methods will continue to be widely used in both existing and new management programs. Therefore, a review of the methods and their assumptions is merited. This article reviews the background of the SDF method and the benefits and approximations inherent in using it. The effect of impermeable boundaries on the SDF is addressed, and a modification to improve SDF-based estimates in narrow alluvial aquifers is presented. Finally, the modified method is demonstrated by comparing it to results from an extensively calibrated numerical model of a managed ground water recharge site.

The Glover Method and Other Analytic Models

Theis (1941) first published a mathematical analysis of transient stream depletion from pumping, providing an integral equation to be evaluated with an infinite-series approximation. Glover and Balmer (1954) provided the analysis with results in the form of an error function. This form of the solution is widely used and referred to as the

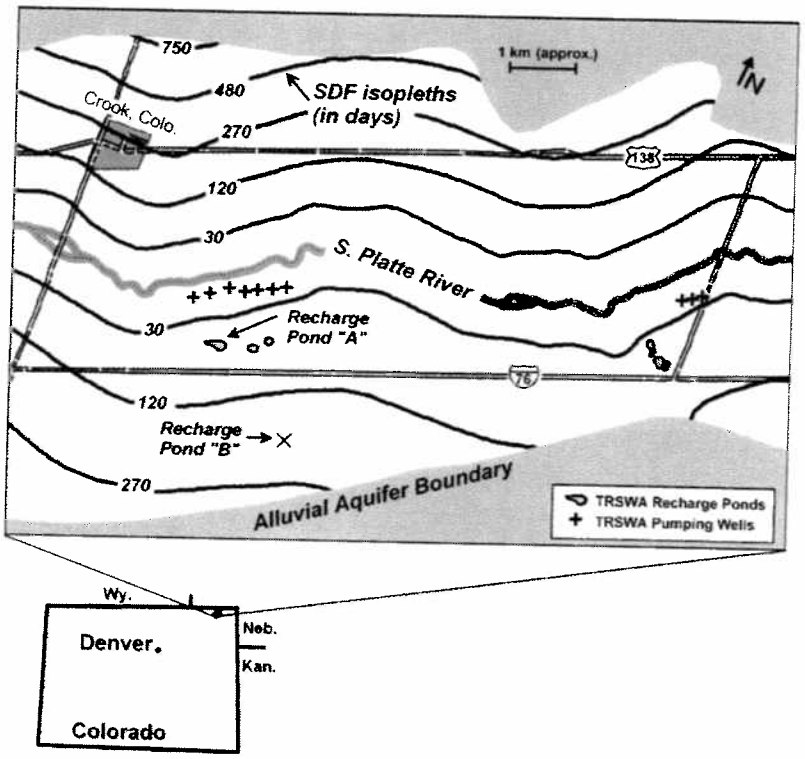

Figure 1. Example SDF map (after Hurr and Schneider 1972a). The SDF is defined as the time when stream depletion volume reaches 28% of pumped volume.

"Glover model" or the "Glover equation." Glover's rate equation (Glover and Balmer 1954) can be written as:

$$\frac{q}{Q} = \operatorname{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) = \operatorname{erfc}\left(\sqrt{\frac{a^2S}{4tT}}\right) \qquad (1)$$

which relates stream depletion rate (q) to aquifer pumping rate (Q) as a function of aquifer properties and time (t). The aquifer properties are the perpendicular distance from the well to the stream (a), storativity (S), and aquifer transmissivity (T). Equation 1 is derived from the Theis's (1935) nonequilibrium equation for drawdown (s) as a function of time and space, $s(x, y, t)$. Zero drawdown along a line at $x = 0$, representing the river, is simulated mathematically by summing the drawdown of the real well with an image well of opposite sign located at an equal distance across the river (at $x = -a$). A specific discharge is calculated from the gradient at and perpendicular to the river ($\partial s/\partial x$ at $x = 0$) multiplied by the transmissivity, and the product is integrated along the river from $y = -\infty$ to ∞ (Theis 1941; Glover and Balmer 1954; McWhorter and Sunada 1977). Theis (1941) considered the most relevant assumptions to be a homogeneous, isotropic semi-infinite aquifer of constant saturated thickness. The river is idealized as a straight line and having "free communication" (Theis 1941) with the aquifer. Integrating Equation 1 gives the following:

$$\frac{v}{Qt} = \left(\frac{a^2}{2tT/S} + 1\right)\operatorname{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) - \left(\frac{a}{\sqrt{4tT/S}}\right)\left(\frac{2}{\sqrt{\pi}}\right)\exp\left(\frac{-a^2}{4tT/S}\right) \qquad (2)$$

which relates cumulative stream depletion volume (v) to cumulative pumped volume (Qt) (Glover 1960; Jenkins 1968a). These relationships are plotted on Figure 2.

Hantush (1955) noted that Glover's solution was also valid in unconfined aquifers for drawdown up to 25% of aquifer thickness, and he generalized the solution to apply to a leaky confined aquifer. Glover (1960) presented the integrated form of the rate equation and later (Glover 1966, 1968, 1978) discussed example applications, including combining it with the method of images for aquifer boundaries. Hantush (1965, 1967) provided depletion analyses for cases with a semipervious streambed and for right-angle bends in the stream.

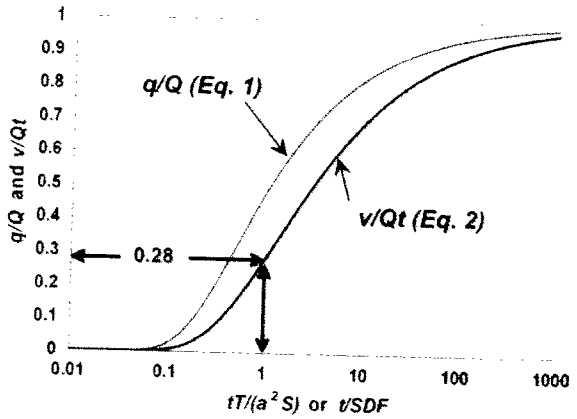

Figure 2. Ideal response curves for stream depletion rate and volume.

These analyses were developed for aquifers that reasonably conform to some combination of mathematically ideal conditions such as fully penetrating streams, permeable streambeds, and semi-infinite, homogeneous isotropic aquifers. Investigators have evaluated these methods for less-ideal field conditions (e.g., Sophocleous et al. 1988, 1995; Spalding and Khaleel 1991; Kollet and Zlotnik 2003), and in recent years, several new analytical models have been advanced to better account for such field conditions (e.g., Hunt 1999, 2003; Fox et al. 2002; Zlotnik 2004; Lough and Hunt 2006). Butler et al. (2001) presented an analytical solution that accounts for the effect of nearby impermeable boundaries, an effect that is the primary consideration of this SDF review. For stream and ground water interactions beyond the scenarios that can be described by these models, Bouwer and Maddock (1997) and Sophocleous (2002) present discussions. This list is not exclusive; a full review of the state of the science in stream depletion is beyond the scope of this article.

This article does not intend to advocate between old or new methods, nor among the many new models. Rather, the motivation of this article comes from the observation that approximations in the SDF method are often overlooked in practice. Based on our ongoing work in the South Platte River basin, the most significant source of error in this setting is due to the impermeable–aquifer boundary effects. This is especially true in narrow alluvial aquifers such as along the South Platte River, where it has been widely used historically and continues to be used today.

The SDF Method

Definition

Jenkins (1968a) provided a review of analytical stream depletion models with the stated purpose of presenting computations in a format that was simple and targeted toward the average user, noting that the equations he presented were not new but "seem to have been rather well concealed from most users." Jenkins also emphasized that residual effects, meaning pumping-induced depletion occurring after pumping has ceased, are often greater than the effects that occur during pumping. He provided explicit examples computing intermittent, residual, and cumulative pumping effects, along with providing user-friendly charts for their computation.

The SDF is defined by Jenkins (1970) as "the time coordinate of the point where $v = 28\%$ of Qt on a curve relating v and t." In a mathematically ideal aquifer, the SDF time can be calculated as a^2S/T since $v/Qt = 0.28$ when $tT/a^2S = 1$ on the nondimensional response curve (Figure 2), but it is important to note that in a nonideal system, the SDF is dependent on additional factors. The SDF was defined to serve as a convenient reference point "anchoring" nonideal response curves to the mathematically

ideal response curve, but the value of 28% is arbitrary and "has no special significance" in itself (Jenkins 1970; Jenkins and Taylor 1974).

Jenkins (1968a, 1968b) created the term SDF to serve as a simple but useful input to the Glover equations, the optimal value of which would be determined through numerical modeling of a complex site. Following convention (Warner et al. 1994), the Glover method refers to using Equation 1 or 2 with the inputs a, S, and T, and the SDF method refers to using the model-derived SDF parameter as the input instead. Writing Equations 1 and 2 as a function of SDF results in Equations 3 and 4, respectively:

$$\frac{q}{Q} = \operatorname{erfc}\left(\sqrt{\frac{\mathrm{SDF}}{4t}}\right) \tag{3}$$

$$\frac{v}{Qt} = \left(\frac{\mathrm{SDF}}{2t} + 1\right)\operatorname{erfc}\left(\sqrt{\frac{\mathrm{SDF}}{4t}}\right) - \left(\sqrt{\frac{\mathrm{SDF}}{4t}}\right)\left(\frac{2}{\sqrt{\pi}}\right)\exp\left(-\frac{\mathrm{SDF}}{4t}\right) \tag{4}$$

Utility of SDF Maps

Jenkins (1968a) is a widely cited reference for the SDF method since this is where the SDF term is first defined and where its intended use was first mentioned. However, the definition and utility are better clarified in subsequent publications (Jenkins 1968b, 1970; Moulder and Jenkins 1969; Jenkins and Taylor 1972, 1974). Jenkins (1968b) provided the first details about the determination of SDF values and discusses the extent of approximation errors he observed when using the method.

Numerical modeling determines stream response curves that take into account the effects of boundaries and other nonideal complexities. SDF values for specific locations in space are determined that best match numerically derived stream response curves to Equation 4 at the point where $v/Qt = 0.28$. The SDF then serves as a single descriptor of complex flow behavior. The basic premise is that stream response curves in complex stream-aquifer systems can be represented by the mathematically ideal Glover equation; i.e., the shape of a nonideal response curve is similar to the ideal curve shown in Figure 2. However, this is not always the case. While all response curves do match at the point where t/SDF = 1 (by definition), nonideal curves may have shapes that deviate from the ideal curve at other points in time.

Jenkins recognized that nonideal curves have different shapes. However, in 266 simulations of locations in the Arkansas River valley in Colorado, he found that most curves matched reasonably well in the range $1/2 < t/\mathrm{SDF} < 2$ (Jenkins 1968b). The error due to the differences between ideal and modeled response curves was considered worth the efficiency gained: complex numerical modeling would be required only once to develop an SDF map. The map could then be used to evaluate multiple pumping management plans (Jenkins 1968b, Jenkins and Taylor 1974). The SDF method was more feasible than repeated modeling (especially with limited computer resources at that time) while still improving the accuracy of computations at locations not easily characterized by analytic methods. This was the real utility of the proposed method.

Stream Depletion in Bounded Aquifers

Glover's analysis assumes an aquifer of semi-infinite extent; i.e., the impermeable boundary lies far enough away from the well that its effects are negligible. For many well locations, however, this boundary has a significant effect. The effect can be accounted for by combining the Glover equation with the image well method. The infinite-series image well pattern for this special case is shown in Figure 3. This is based on the more general image well pattern used for computing drawdown from pumping between a river and an impermeable boundary (Ferris et al. 1962; McWhorter and Sunada 1977), which includes image wells across both the river and the impermeable boundary. In the case of stream depletion, however, Glover's equation already encompasses the image wells across the river, so image wells have to be added only for the impermeable boundary (see Appendix).

Using Equation 2 with the image pattern shown in Figure 3, we constructed response curves for a number of bounded (but otherwise ideal) aquifers with various widths (W), hydraulic diffusivities (T/S), and well locations (Figure 4). These curves were verified with MODFLOW (McDonald and Harbaugh 1988; Harbaugh et al. 2000). Here, W is defined as the distance between the river and the impermeable boundary. The resulting nondimensionalized response curves are a function only of well position with respect to the stream and aquifer boundaries, i.e., the ratio a/W. Note again that by definition all curves match at the point where t = SDF. For wells close to the river, the difference between ideal and bounded curves is small and not apparent until well after the SDF time. For wells closer to the impermeable boundary, the differences are larger. Differences between the ideal curve shape and the various bounded curves are also plotted in Figure 4 as percent error.

Nonideal Conditions Accounted for in the SDF

Using the Glover method with image wells accounts for impermeable boundary effects but does not address other nonideal conditions. In contrast, Jenkins and Taylor (1974) note that the SDF is "a value of time that reflects the integrated effects of the following: irregular impermeable boundaries; stream meanders; aquifer properties and their areal variations; distance to the point from the stream; and imperfect hydraulic connection between the

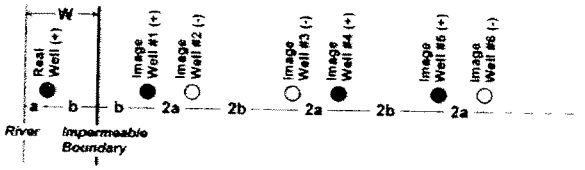

Figure 3. Infinite-series image well pattern for stream depletion analysis near an impermeable boundary.

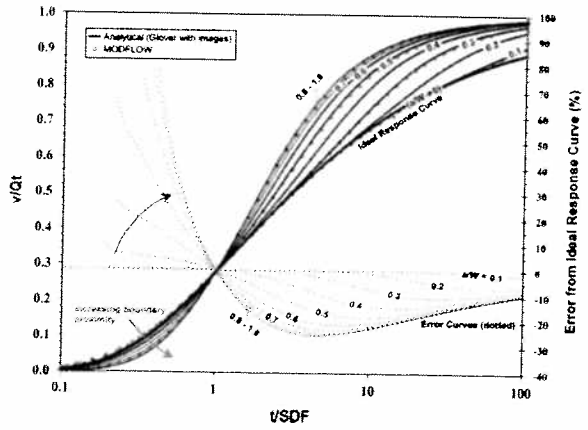

Figure 4. Response curves for bounded (but otherwise-ideal) aquifers.

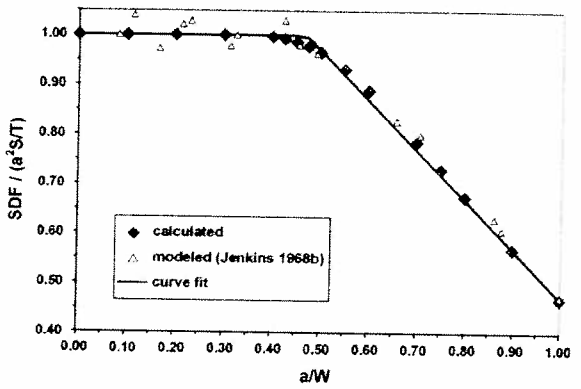

Figure 5. SDF as a function of *a/W*.

stream and the aquifer," and they suggest that all these factors could be taken into account in the SDF parameter, with the level of detail for any particular modeling project chosen to suit the project's particular needs and available computing capacity.

Documentation of the numerical modeling used in generating the USGS SDF maps (e.g., Hurr and Schneider 1972a, 1972b) is limited, but based on descriptions in Moore and Wood (1967), Jenkins (1968a, 1968b), Moulder and Jenkins (1969), Hurr and Schneider (1972b), Jenkins and Taylor (1972, 1974), Missouri Basin States Association (1982), Warner et al. (1994), and the recollection of USGS personnel (R.R. Luckey, personal communication, 2005), the maps account for spatially variable transmissivity, the presence of stream and aquifer boundaries and their irregular shapes (e.g., meandering streams and the presence of tributaries), and the location of the well with respect to the stream and aquifer boundaries.

Boundary Effects in the SDF

Because SDF maps account for other nonideal conditions in addition to boundaries, we propose that the maps provide a useful description of alluvial aquifer behavior. With modification, the SDF method can be combined with the image method to improve estimates in bounded aquifers while still benefiting from the integration of nonideal effects provided by the numerical modeling used to generate the maps. The procedure proposed is to remove the effect of impermeable boundaries from modeled SDF values and then combine this modified SDF with image wells to account for impermeable boundaries. Modifying the SDF before use with images prevents "overaccounting" for impermeable boundary effects.

Jenkins (1968b) observed in a series of numerical model tests that SDFs approximately equaled a^2S/T for locations where $a/W < 0.5$, but SDFs were less than a^2S/T when the well was located nearer the impermeable boundary ($a/W > 0.5$). Figure 5 shows the ratio of SDF to a^2S/T as a function of a/W. This plot was constructed analytically and is similar to numerical model results plotted by Jenkins (1968b). Model results from Jenkins (1968b) are also included in Figure 5. Not included are results from within a distance of four model nodes from the river, which Jenkins noted as being scattered and having the (SDF)/(a^2S/T) ratio greater than 1 due to the finite-difference approximation.

The (SDF)/(a^2S/T) ratio is approximately 1 for wells near the stream. This can be demonstrated by assuming that the boundary effect is insignificant when $v/Qt < 0.005$ for the closest boundary image well. This corresponds to $q/Q < 0.021$. Setting Equation 1 equal to 0.021, the argument of the complementary error function for the first image well, $\left[r_i^2 S/(4Tt)\right]^{1/2}$, equals 1.632, where r_i is the distance from the river to the first image well. At the SDF time, $t = a^2S/T$, so substitution yields $r_i/a = 3.264$. Accounting for the boundary location with respect to r_i yields $a/W = 0.47$. Thus, for wells located at $a/W < 0.47$, the boundary will have a negligible effect when $t/\text{SDF} \leq 1$. Knight et al. (2005) made a similar observation. The effect of the impermeable boundary on stream depletion can still be significant for pumping located near the stream, but only for time greater than the SDF. This is true even if pumping ceases before the SDF time, since depletions continue after pumping ceases.

Figure 6 shows an example of response curves for $a^2S/T = 100$ d and $0.1 < a/W < 1.0$. These curves were constructed using Equation 2 with image wells (Figure 3) and verified with MODFLOW (McDonald and Harbaugh 1988; Harbaugh et al. 2000). For all locations where $a/W \leq 0.50$, the SDF (defined as the time when $v/Qt = 0.28$)

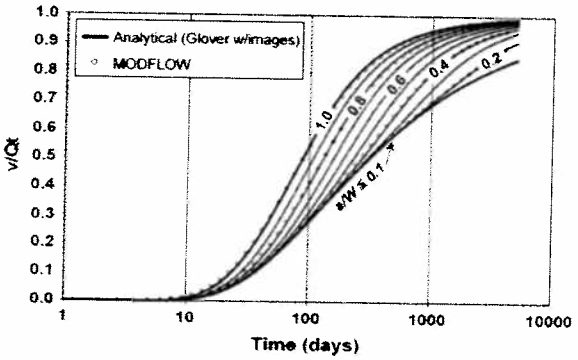

Figure 6. Example of depletion response curves for a^2S/T = 100 d and $0.1 < a/W < 1$, showing effect of impermeable boundary lowering SDF to less than a^2S/T.

occurs at very close to 100 d. The effect of the boundary is significant only at larger times. Locations where $a/W > 0.5$ are affected earlier; for example, $v/Qt = 0.28$ at 80 d for $a/W = 0.7$. This analysis demonstrates that SDFs for the area closer to the river than to aquifer boundary can be used directly with the method of images without concern for boundary effects already reflected in the SDF. In the other portion of the aquifer, the boundary effect in the SDF follows a predictable pattern (Figure 5), allowing the effect to be easily removed.

Example of Impermeable Boundary Effects in Stream Depletion

Managed Ground Water Recharge for River Augmentation

SDF values can be combined with the image method by first removing the effect of the impermeable boundary. This allows the user to more accurately account for these boundaries using image wells while retaining other aquifer information such as spatially variable transmissivity incorporated into SDF values. This approach was tested against a numerical model (MODFLOW-2000, Harbaugh et al. 2000) constructed for the Tamarack Ranch State Wildlife Area (TRSWA) recharge project (Halstead and Flory 2003). The model has been extensively calibrated under transient conditions using hydraulic heads from 35 wells at 13 different time periods.

The TRSWA recharge project is located adjacent to the South Platte River in northeast Colorado. The project currently includes 10 high-capacity wells located near the river that divert water to a series of recharge ponds located approximately 1200 m from the river. These locations are noted on Figure 1. Current annual recharge capacity is approximately 7 million m^3, with a future design capacity of 12 million m^3. The system is operated in periods of low demand on the river, typically winter and early spring, with the pumping schedules and recharge pond locations designed to increase ground water discharge to the river during the high-demand summer months. The TRSWA recharge project is one part of an overall ground water retiming effort designed to provide extra water to the Platte River during times of shortage to benefit Colorado native species and help meet flow-related goals of the Platte River Recovery Implementation Program under the Endangered Species Act.

More than 4000 wells pumping from the South Platte alluvium rely heavily on similar projects as a source of river augmentation water. This system of managed ground water recharge redistributes water in time, enabling junior ground water appropriators to withdraw water from tributary alluvial aquifers during high-demand periods without harming existing senior surface water rights holders. Annual managed ground water recharge to rivers in Colorado increased from 24 million m^3 in 1980 to 220 million m^3 in 2005.

Assessing the timing and volume of river depletions and augmentation from such pumping and recharge operations is typically performed with the Glover or SDF method. The errors plotted in Figure 4 might be considered small for many ground water applications, but the potential for errors up to 30% is of considerable concern in water rights administration. Also, even while the volume error may be small at a given time, the predicted time for a given volume (for example, the volume recharged during one season of operation) to discharge back to the river may be significantly longer if boundary effects are not accurately accounted for in the estimate. This has other important ramifications in water rights administration. For example, in Colorado, ground water from "augmentation wells" is sometimes pumped directly to the river to replace out-of-priority surface water diversions or depletions. In this situation, properly accounting for boundary effects can make the difference between the augmentation well depletions being assumed to spread over many years and accumulating with each additional season of operation, or possibly having the depletion assessment limited to just one or two seasons.

Combining the SDF Method with Image Wells

Boundary effects were found to be significant for the TRSWA recharge project since it is located in a narrow portion of the alluvial aquifer (2000 m < W < 3500 m) and the timescales of interest for the project site are much larger than the SDFs of the recharge pond locations. Two locations at the TRSWA recharge project were selected for illustrating the adjustment of SDFs for boundary effects. Recharge pond A has a mapped SDF of approximately 60 d and an aquifer position of $a/W = 0.31$. A hypothetical recharge pond B was selected at a location nearer the impermeable boundary and has a mapped SDF of approximately 200 d and an aquifer position of $a/W = 0.79$. These SDF values and boundary distances were obtained from the local USGS map (Figure 1). Interpolation between SDF isopleths is done using the square root of SDF values since SDF is proportional to a^2 (Jenkins 1968b). The mapped values (SDF_{USGS}) were modified ($SDF_{modified}$) by the following equation based on the simple linear fit shown in Figure 5:

$$\begin{aligned} SDF_{modified} &= SDF_{USGS} && (a/W<0.47) \\ &= SDF_{USGS}(1.47 - a/W)^{-1} && (a/W>0.47). \end{aligned} \quad (5)$$

Comparisons between the numerical model and the modified SDF method for recharge ponds A and B are shown in Figures 7 and 8, respectively. Figure 7 shows that for pond A the SDF method without images compares well with the numerical model until times much larger than the SDF. Adding image wells improved the fit at the later times, and no modification to the SDF value was needed for use with images at this location. For pond B, the numerically modeled response curve and the SDF method without image wells differ significantly when $t >$ SDF. Adding image wells to the unmodified SDF provides significantly better agreement with the numerical model, but it overpredicts discharge to the river. Using the modified SDF value (Equation 5) with images improved agreement with the model.

It is interesting that the SDF values obtained from the 1972 USGS map (Hurr and Schneider 1972a)

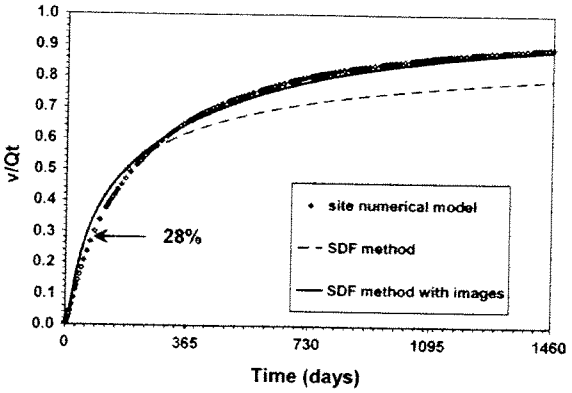

Figure 7. Comparison of analytic response curves to numerical model results for recharge pond A ($a/W = 0.31$, SDF = 60 d).

provided a good match to the more detailed and calibrated recent site-specific model. In this case, the SDF maps appear to provide a reasonably accurate reference for water rights computations.

Summary

This article reviews the background of the SDF method and its behavior near impermeable aquifer boundaries. New stream depletion response curves were provided that can be used to assess such boundary effects. The need to refine stream depletion analyses in bounded alluvial aquifers can be significant. Combining the image method with SDFs without overaccounting for boundary effects was discussed: mapped SDF values closer to the stream than to the impermeable aquifer boundary can be used without modification since boundary effects are insignificant in the SDF there, and SDF values in the other portion of the aquifer can be used with image wells by first removing the impermeable boundary effect from the SDF. This allows the user to better account for alluvial aquifer boundaries while retaining other nonideal aquifer effects integrated into mapped SDF values.

A modified SDF method is still an approximation since, like boundary effects, other nonideal complexities are exhibited to different degrees at the time of the SDF. Still, the proposed method compared well to results obtained with a detailed and extensively calibrated numerical model of a ground water recharge site. It could be a useful alternative when it is not feasible to use numerical modeling for water management decisions. The modified method can improve stream depletion estimates while retaining the value of SDF maps as references for water rights administration.

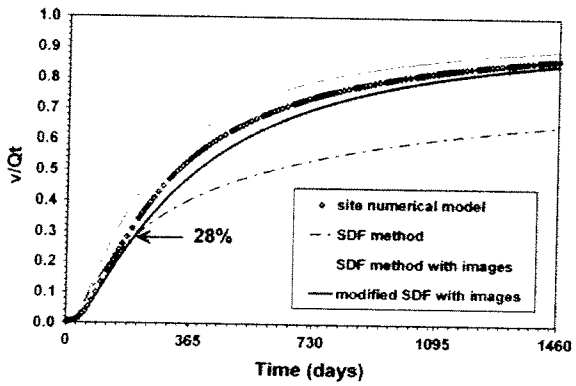

Figure 8. Comparison of analytic response curves to numerical model results for recharge pond B ($a/W = 0.79$, SDF = 200 d).

Acknowledgments

Funding was provided in part by the U.S. Department of Agriculture Cooperative State Research, Education and Extension Service National Research Initiative, Award No. 2002-35102-12471. The contributions of three anonymous reviewers improved the article and were appreciated.

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Appendix

The use of image wells to represent flow boundaries is certainly well understood by ground water professionals. Still, to our knowledge, the particular series appropriate for use with the Glover stream depletion equation in a bounded alluvial aquifer (Figure 3) does not appear in commonly available references. Therefore, computational approaches are presented here for reference.

Stream depletion flux in a bounded aquifer is given by Equation 1 and the first four image wells in the infinite series (Figure 3) as:

$$\frac{q}{Q} = \operatorname{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) + \operatorname{erfc}\left(\frac{a+2b}{\sqrt{4tT/S}}\right) - \operatorname{erfc}\left(\frac{3a+2b}{\sqrt{4tT/S}}\right) - \operatorname{erfc}\left(\frac{3a+4b}{\sqrt{4tT/S}}\right) + \operatorname{erfc}\left(\frac{5a+4b}{\sqrt{4tT/S}}\right) + \ldots \quad \text{(A1)}$$

where b is the distance between the well and the aquifer boundary; i.e., $b = W - a$, and other variables are as defined previously.

Knight et al. (2005) provided a summation equation for this scenario, which is convenient for programming in mathematical software when a large number of images are required:

$$\frac{q}{Q} = \operatorname{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) + \sum_{n=1}^{i}\left[(-1)^{n+1}\left(\operatorname{erfc}\left(\frac{2nW - a}{\sqrt{4tT/S}}\right) - \operatorname{erfc}\left(\frac{2nW + a}{\sqrt{4tT/S}}\right)\right)\right] \quad \text{(A2)}$$

For stream depletion volume (e.g., in constructing Figures 4 and 6), we arranged Equation 2 and the image well pattern (Figure 3) into the following summation:

$$\frac{v}{Qt} = \sum_{n=0}^{i}[(-1)^n C + (-1)^n D] \quad \text{(A3)}$$

where

$$C = \left(\frac{(2nW+a)^2}{2tT/S} + 1\right)\operatorname{erfc}\left(\frac{2nW + a}{\sqrt{4tT/S}}\right) - \frac{2nW + a}{\sqrt{4tT/S}}\frac{2}{\sqrt{\pi}}\exp\left(-\frac{(2nW+a)^2}{4tT/S}\right)$$

and

$$D = \left(\frac{(2W+2nW-a)^2}{2tT/S} + 1\right)\operatorname{erfc}\left(\frac{2W + 2nW - a}{\sqrt{4tT/S}}\right) - \frac{2W + 2nW - a}{\sqrt{4tT/S}}\frac{2}{\sqrt{\pi}}\exp\left(-\frac{(2W+2nW-a)^2}{4tT/S}\right)$$